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Drumlin relief

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ABSTRACT

Drumlin relief is a key parameter for testing predictions of models of drumlin formation. Although this metric is commonly described in textbooks as being of the order of a few tens of metres, our critical review of the literature suggests an average value of about 13 m, but with much uncertainty. Here we investigate a large sample of drumlins (25,848) mapped from a high resolution digital terrain model of Britain, which allowed the identification of extremely shallow drumlins. Results indicate that most drumlins have a relief between 0.5 and 40 m (with a surprisingly low average value of only 7.1 m) a mode of 3.5–4 m, and with 41% of all drumlins characterized by a relief <5 m. Drumlin relief is found to never exceed 7% of the width and is positively correlated with this parameter, possibly indicating that drumlins need a large base to stand against the flow of the ice. Drumlin relief is also positively correlated with the length, which shows that drumlins do not grow in length by redistributing sediments from their summits to their downflow (lee) end, as previously hypothesised.

1. Introduction

Drumlins may be loosely defined as oval-shaped hills that are found in terrains formerly occupied by an ice sheet and whose shape is aligned in the direction of ice flow. Despite most drumlins being characterized by a similar, typical shape, their structure and internal composition is found to be diverse and includes at least three end members: bedrock-cored, till, and stratified sediments (Stokes et al., 2011). This has led to the formulation of several competing hypotheses on the formation of drumlins, variously based on erosional, depositional, and deformational processes (or a combination of the three) caused by flowing ice and/or by subglacial meltwater (e.g., Fairchild, 1929; Smalley and Unwin, 1968; Shaw and Freschauf, 1973; Boulton, 1987; Shaw, 1989; Dardis and Hanvey, 1994; Hindmarsh, 1999; Fowler, 2000; Aber and Ber, 2007). As drumlin formation is still far from being unequivocally deciphered, these landforms remain amongst the most enigmatic features in geomorphology. One possible contribution to this unsolved problem is the analysis of drumlin morphometry. Insights about drumlin formation processes might come from a better knowledge of their size and shape properties and what controls the spatial variation of these metrics within a drumlin field. At the very least, a quantitative characterisation of drumlins may be used to formally test various formation theories, some of which have already been developed into numerical models (e.g., Hindmarsh, 1999; Fowler, 2000; Pelletier, 2008).

The simplest view of drumlins is that they are merely bumps in a landscape, and one of the most fundamental aspects of such a landform is therefore its relief (height), defined here as the distance from the top of the landform to its base. Other metrics, (such as drumlin length, width, elongation, and shape) have recently received renewed attention with studies based on unprecedentedly large databases (Clark et al., 2009; Spagnolo et al., 2010, 2011). However, in order to complete our understanding of drumlin size, a comparably comprehensive analysis on drumlin relief is needed. Although the literature on drumlin relief, as discussed in the following section, is considerable, statistically robust analyses of this metric over large databases are extremely rare (e.g., Hättestrand et al., 2004).

A series of open questions about drumlin relief appears to be particularly relevant to a better understanding of drumlin formation. Drumlin relief is commonly (e.g., from textbooks, see literature review below) perceived as of the order of a few tens of metres. Why are not drumlins any 'taller'? Given thicknesses of ice exceeding many kilometres, and probably no shortage of mobilised sediment beneath ice sheets (e.g., Nygård et al., 2007), process reasons or bounds must be preventing drumlins from growing up to many hundreds of metres in relief. Does something limit their height? And what is the minimum value of drumlin relief? Could new technologies and higher resolution data reveal a lower boundary than previously found? What is the relationship between drumlin relief and other metrics? Is it possible to confirm that drumlins become progressively longer through a redistribution of sediment from their summit to their downflow (lee) end (e.g., Piotrowski, 1989)? A first step in addressing these and other questions is a robust quantification of drumlin relief, which we present in this paper. Here, drumlin relief is analysed with GIS techniques from over 25,848

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Table 1
Drumlin relief estimates or measurements drawn from the literature. Sample size is included and minimum, maximum and mean (with standard deviation, SD) reliefs are tabulated. The expression “(est)” stands for estimated value, i.e. a value that might have not been rigorously derived from a statistical analysis of measured values. In the last row the weighted (as per sample size) mean value (13 m) derived from the above values in the cb is reported, based on a sample of 13,427 drumlins spread over 6 countries.

Paper	Location	No. of drumlins	Relief (*h* in m)		
			Min	Mean	Max
Davis (1884)			6 (est)		76 (est)
Hubbard (1906)	New York State, USA	75	6		30
Fairchild (1907)	New York State, USA	31			67
Armstrong and Tipper (1948)	British Columbia, Canada			15–23 (est)	46 (est)
Bird (1953)	NW territories, Canada				6 (est)
Sharp (1953)	Minnesota, USA	10s	3		8
Kupsch (1955)	Saskatchewan, Canada	98	6		46
Colton and Lemke (1955)	North Dakota, USA		1 (est)	3 (est)	8 (est)
Wright (1957)	Minnesota, USA			5–10 (est)	
Aronow (1959)	North Dakota, USA	20			15
Gluckert (1973)	Pieksämäki and Keitele, Finland	14,500	10		120
Gravenor (1974)	Nova Scotia, Canada	150	8	18 (est.)	30
Trenhaile (1975)	Galt-Hamilton, Ontario, Canada	500	7.5		38
	Guelph, Ontario, Canada				
Jauhiainen (1975)	Klaipéda, Lituania	99	2.5	4.4 (SD 1.3)	32.5
	Doboprole/Przybiernów, Poland	76		8.7 (SD 4.8)	
	Wierzbięcin/Dzwonowo, Poland	261		7.5 (SD 3.3)	
	Stargard Szczeciński, Poland	131		8.2 (SD 4.2)	
	Kórnik/Zaniemýsl, Poland	90		5.5 (SD 2.6)	
	Šmigiel, Poland	82		8.5 (SD 6.4)	
	Ober-Ücker/Greiffenberg, Germany	90		10.2 (SD 3.7)	
	Brüssow, Germany	55		8.3 (SD 3.7)	
	Lake Lieps, Poland	50		9.7 (SD 4.1)	
	Rosenow, Germany	52		6.3 (SD 2.5)	
	Bad Oldesloe, Germany	117		8.3 (SD 4.7)	
Mills (1980)	various locations, USA	55 blocks 25 dr. each		4–26, mode 14–16	
Krüger and Thomsen (1984)	Iceland	10s	1		3
Sharpe (1985)	NW Territories, Canada	3	10		25
Zakrzewska Borowiecka and Erickson (1985)	Wisconsin, USA	3893	3	13 (SD 7)	45
Harry and Trenhaile (1987)	Ontario, Canada	315		12 (SD 7.1)	
Rabassa (1987)	J. Ross Island, Antarctica	2	7		12
Clapperton (1989)	Patagonia, Chile	10s	4	20–25 (est)	50
Rouk and Raukas (1989)	Estonia	1000	15		63
Boyce and Eyles (1991)	Ontario, Canada	998			60
Francek (1991)	New York State, USA	3984		13.5 (9.4 SD)	
Grosswald et al. (1992)	East Siberia, Russia		10 (est)		30 (est)
Wysota (1994)	Koziary, Poland	138	1.5	3.4	8
	Gorzno, Poland	59	2	4.8	13
	Trepki-Samin, Poland	134	1.5	3.8	18
	Janowko, Poland	186	1.5	4.8	22
Menzies (1996)	Scotland, United Kingdom	160	3	13.3 (7.8 SD)	37
Zelcs and Dreimanis (1997)	Latvia	1430		6–12 (est)	35
Rattas and Kalm (2001)	Estonia	100		20–40 (est)	
Jørgensen and Piotrowski (2003)	Denmark	161		5–10 (est)	15
Hättestrand et al. (2004)	Sweden	3280		16	
Smith et al. (2009)	Scotland, UK	175	1.7	9.3 (7 SD)	65
Johnson et al. (2010)	Iceland	>50	5		10
Weighted mean value		13,427		13.0	

landforms mapped from a high resolution digital terrain model (DTM) of Britain. The aim is to test if drumlins:

(i) are limited to a certain relief range;
(ii) are mostly formed at a certain and preferred relief; and
(iii) show any relationship between relief and other metrics (e.g., the length).

2. Literature review

The literature on drumlins is vast and spans over three centuries (Menzies, 1984). Of more than 400 papers, at least 33 provide a quantitative statement about drumlin relief (Table 1). Investigations cover most of the formerly glaciated world, with studies in South (e.g., Clapperton, 1989) and North America (e.g., Francek, 1991), Fennoscandia and northern Europe (e.g., Hättestrand et al., 2004), and Antarctica (e.g., Canals et al., 2000).

In an attempt to derive a quantitative estimate of drumlin relief from the literature, all 33 papers in Table 1 have been scrutinised in detail. Unfortunately, we soon discovered that some of them appear to be based, at least in part, on generic estimates rather than direct measurements. Assertions such as “[drumlins] vary considerably in height, probably averaging between 50 and 75 feet, although many are lower and a few range up to 150 feet high” (Armstrong and Tipper, 1948, p. 289) are not rare. Some papers do not specify the technique applied to analyse drumlin relief, and a statistically evaluated average relief value is often missing (Table 1). We also noticed that only 23 of the 33 mentioned papers indicated the overall number of drumlins considered in the analysis. When the sample size is mentioned, it typically appears to be limited to a few hundreds of drumlins, although at least three important exceptions exist (Gluckert, 1973; Francek, 1991; Hättestrand et al., 2004). Because of all these caveats, it is difficult to judge how representative a quantitative estimate of drumlin relief from the literature could be. In general, drumlin relief appears to range from 1 (Krüger and

Thomsen, 1984) to 120 m (Gluckert, 1973), while mean reported values range from 4 (e.g., Wysota, 1994) to 26 m (Mills, 1980). Whenever a statistical value of mean drumlin relief was indicated in a paper, the value was used to evaluate a global weighted mean relief based on all studied drumlins from all the papers (last row in Table 1). In this way, we assembled a sample size of 13,427 drumlins, producing a weighted mean relief of 13 m. This value still needs to be treated with caution given the variety of investigators and methodologies and map scales used, and it is worth noting that Rose and Letzer (1975) highlighted how drumlin relief estimates from topographic maps could be flawed. However, it is interesting to notice that the weighted mean relief of 13 m is surprisingly low given how drumlins are normally described in textbooks. Here, drumlin relief is often reported in terms of generic value (e.g. several tens of metres, in Ahnert, 1998), maximum values (e.g. up to 60 m, in Embleton and King, 1975), or relief range (e.g. 20–30 m, in Tricart, 1970; or 5–50 m in Summerfield, 1991 or Bennett and Glasser, 1996), but always giving the overall impression that drumlins are landforms of a few tens of metres height.

Only a few papers analyse the actual variability of drumlin relief (i) in relation to other morphometric variables such as length or width (Menzies, 1979; Mills, 1980; Shaw and Kvill, 1984; Harry and Trenhaile, 1987; Mills, 1987), (ii) in respect to inferred drumlin age (Hättestrand et al., 2004; Smith et al., 2009), and (iii) spatially within an ice lobe (Francek, 1991). Those who attempted a correlation analysis between drumlin relief and other measures such as length, width, and elongation, all found that the correlations were generally weak. The only exception was the correlation between drumlin relief and width, found to be generally good in the Glasgow area in the UK (Menzies, 1976) and in several localities of the US (Mills, 1980) and in Ontario, Canada (Harry and Trenhaile, 1987). In his discussion on this, Mills (1980, p. 639) concluded that "...the drumlin nucleus is streamlined by the accretion of till, with the bulk of the accretion occurring as a "tail" in the lee of the nucleus. The length of the tail is determined chiefly by how constant the direction of the ice flow remains during and after drumlin formation. Drumlin width and height are determined by other, still undetermined, factors." And later on (Mills, 1987), he suggested that width, and possibly relief, values may reflect a relatively narrow, yet unspecified, range of mechanical properties of ice or substrate that are associated with drumlin formation.

3. Method for measuring drumlin relief

In his book *The Quaternary Era*, Charlesworth (1957) commented that drumlin relief is not so commonly recorded because it is not easily judged in the field or read from maps. In fact, measuring drumlin relief, especially on a large population, is a time-consuming process in the field; and although faster when derived from topographic maps, methodological issues mostly related to the vertical resolution of the maps exist (see Rose and Letzer, 1975). In this paper, we overcome these issues by applying a GIS-based method for automatically measuring drumlin relief using two data sources: a high resolution DTM of land surface elevations and a map of drumlin outlines.

Before detailing our GIS methods for deriving data, we first reflect on what is actually meant by drumlin relief. Although we have not found any explicit discussion of the matter, we note that one could view drumlins as discrete, blister-like features (or half eggs) superimposed on the landscape or as waveforms that are part of a continuous surface (i.e. the drumlin field) (Fig. 1), like waves in the sea. This latter view is, in particular, what some modellers tend to favour (e.g., Hindmarsh, 1999; Fowler, 2000). We do not know which of these options is the most appropriate model for drumlins, although it seems that the related bedforms of ribbed moraine (e.g. Dunlop and Clark, 2006) and mega-scale glacial lineations (e.g. Clark, 1993; Canals et al., 2000) fairly convincingly appear as waveform phenomena (i.e. continuous undulating surfaces rather than discrete landforms). We also note that researchers currently devising process theories into numerical models tend to view such landforms and drumlins as waveform phenomena and that wavelength and amplitude are appropriate measures. However, examination of most drumlin fields (such as in Fig. 2B) tends to reveal prominent breaks of slope marking drumlin boundaries and with relatively flat (non wave-like) areas in between. That we can clearly observe definable edges to drumlins suggests that either the blister-on-the-landscape model is most appropriate or that we are being fooled into this view as they are actually waveforms with sedimentation filling the hollows (Fig. 1). This is a fundamental and as yet unresolved question about drumlins and which must have great bearing with regard to how they form. We do not tackle this problem here because our method utilises just the land surface with no information on surficial sediments or depths of structures beneath the surface. Hence, we cannot distinguish between the models and are only measuring *h* rather than a possible *H* (in Fig. 1). Even if we suppose, on theoretical grounds, that the waveform model is appropriate, we have no means of measuring this for a large sample. The issue of postglacial sediment infills between drumlins is likely to be more serious in flat and wet places (such as much of Arctic Canada and Fennoscandia) where bogs or mires tend to occur between drumlins, rather than in more undulating terrains where drainage is easier. Fieldwork experience of our target drumlin fields in Britain, and which are mostly now occupied by farmed pastures, tends to

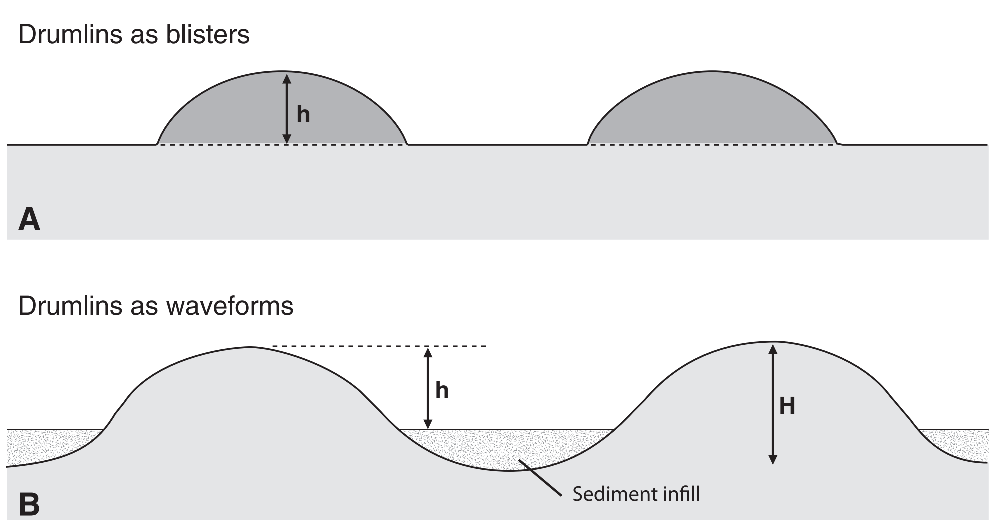
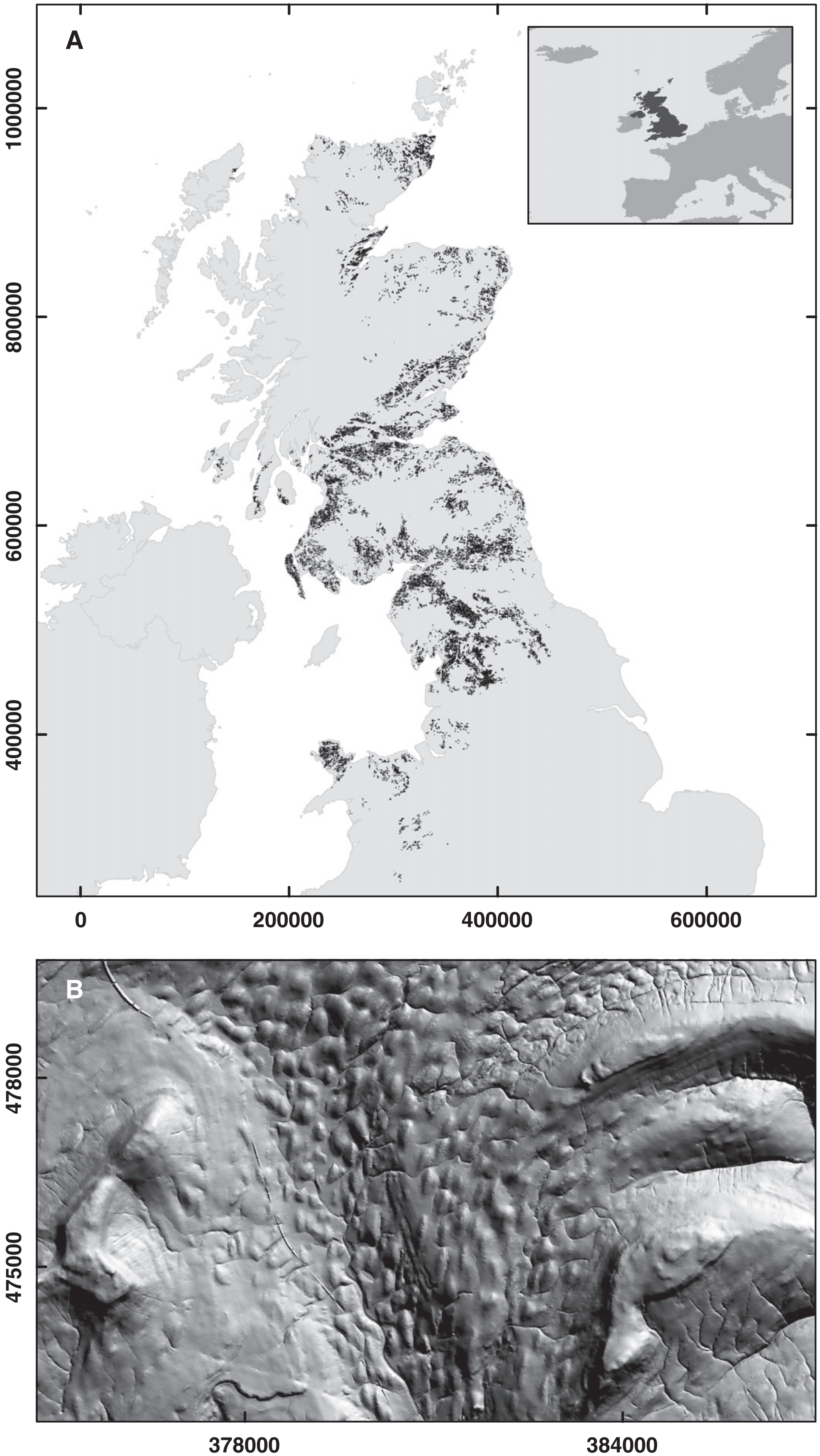


Fig. 1. Two conceptual views of drumlins: as blisters superimposed on the landscape (A) or as waveforms (B) with sediment infill in the hollows, possibly from sedimentation in inter-drumlin bogs or mires. *h* represents the relief of the drumlins as measured in this paper; *H* is the waveform amplitude.


A
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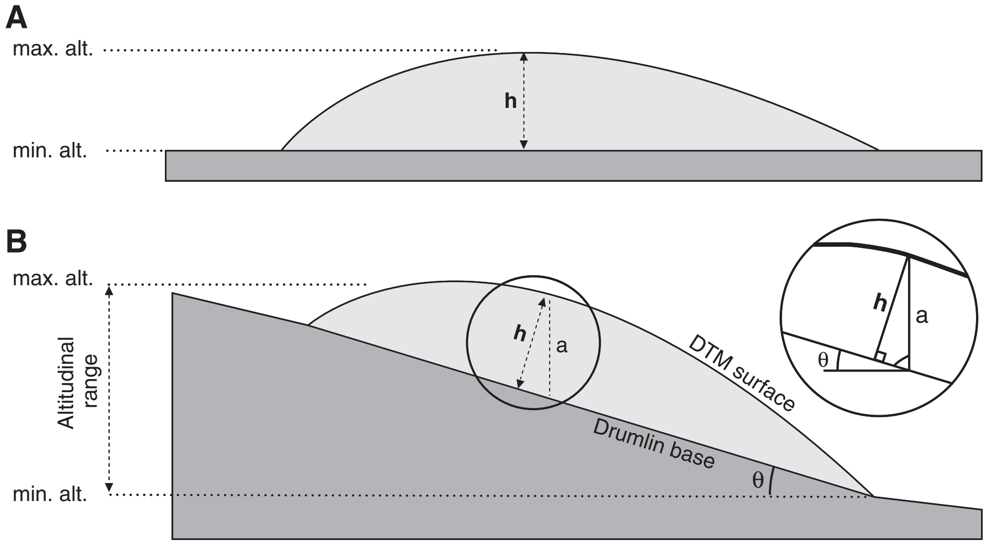

Fig. 3. Definition of drumlin relief for (A) flat and (B) hillslope contexts. Note the difference between the altitudinal range and the real relief when drumlins are found on hillslopes. Our GIS procedure for measuring relief (h) interpolates a drumlin base (see text) and then finds the maximum elevation difference between this and the elevation of the surface DTM to derive the vertical offset (a), which is then converted to drumlin relief using $h = a * \sin(90 - \theta)$, where θ is the slope angle of the apparent drumlin base. The circular inset is a zoom to show the trigonometric properties that connect h to a.

suggest the latter case and that infills should not be too much of a source of error. Nevertheless, this is a caveat to our measurements, which treated with caution should be taken as minimum estimates of drumlin amplitude. An additional caveat is that an originally generated drumlin surface might have been lowered by subglacial erosion or postglacial slope processes or might have been raised by the addition of other sediments blanketing them.

Our work is based on a drumlin mapping project that was carried out using a horizontal 5-m and vertical <1-m resolution DTM available for Britain (NEXTMAP Britain, InterMap Technologies © BGS(NERC)) (Clark et al., 2009; Hughes et al., 2010). The mapping covered all recognisable drumlins in the formerly glaciated area of Britain (England, Scotland, and Wales) (Fig. 2). Each drumlin was directly mapped on-screen into ArcGIS by using analytical hill-shading visualisations of the DTM at two orthogonal illumination orientations and one with the illumination positioned directly overhead, as per Smith and Clark (2005), and with a vertical exaggeration factor of four (methods fully described in Hughes et al., 2010). Drumlins were digitised as smooth outlines (*shapefile* polygons in ArcGIS) following the most evident break of slope demarcating the perimeter of the drumlin. A total of 36,032 drumlins were recognised and mapped. With such a large number, formally or quantitatively check mapping accuracy is difficult, and some mistakes in the interpretation and exact outlining of some landforms likely occurred. However, statistical theory suggests that in order to obtain a good estimate of a population parameter, the size of the sample is more important than the accuracy of the measurement, as long as any error is not systematic. This is certainly the case when the number of samples rises to the tens of thousands. In other words, we judge that small mapping mistakes are unlikely to affect the final result because they would be swamped by the large size of the sample, as long as they are not systematic. Indeed, when mapping accuracy was tested during field visits to some of the mapped areas, it was found to be sufficiently high and in many cases to exceed field-based methods of mapping (see Clark et al., 2009, for further details). More importantly, no systematic mistakes were highlighted during these field checks.

Like triangles in geometry, drumlin relief (height) can be defined as the distance between the top of the landform and its base, measured perpendicularly to the base. In the past, drumlin relief has been measured as the difference in elevation between the point of highest elevation and that of lowest elevation, i.e. the drumlin altitudinal range (Fig. 3A) presuming the blister-on-the-landscape model. Usually this was achieved by counting elevation contours on topographic maps. If a drumlin is expressed as a bump on a flat horizontal landscape, its relief does in fact correspond to its altitudinal range. In Britain though, some (thousands of) drumlins lie on a hillslope rather than on a horizontal surface (e.g. in Fig. 2B). In these cases, the altitudinal range of a drumlin is much greater than the correct relief of the drumlin (Fig. 3).

The method that we adopt, for convenience, assumes that the drumlin base is a planar, but not necessarily horizontal, surface. Drumlin relief is therefore equivalent to the longest line perpendicular to the flat base that can be drawn within the body of a drumlin, h in Fig. 3. The method involves three steps.

(i) *Remove drumlins from the landscape*. From the original DTM (Fig. 4A), a new DTM is generated as if the drumlins were physically sliced off the landscape (Fig. 4B) and replaced by a planar surface in their place (i.e. drumlin surface replaced by drumlin base) (Fig. 4C). To do this, each drumlin outline is sampled every 5 m to extract a series of points around the perimeter of the drumlin. The elevation above sea level of each point is then extracted from the original DTM and a new triangulated irregular network (TIN) is created from these points of known elevation to re-create a planar surface over the area formerly occupied by the drumlins. The TIN is then converted into a new DTM, which represents the conceptual 'drumlin-less' surface.
The technique described here has the advantage of relying on one of the most basic and universal tools in GIS (the TIN construction) that can quickly and easily be applied to a large volume of data. A more sophisticated technique has been recently suggested in order to analyse drumlin relief (Smith et al.,

Fig. 2. Coverage and distribution of all 36,032 mapped drumlins shown as black dots (A) and an analytical hill-shading visualisation of the NEXTMap DTM at Horton in Ribblesdale (Yorkshire Dales, northern England) (B). Here, drumlins are apparent as blister-like features superimposed on the landscape. Note how clear it is to see the perimeter break-of-slopes that demarcate each drumlin and also that some are positioned on the valley floor but with many on the adjacent hillslopes. Ice flow direction was towards the south. Coordinates refer to the British National Grid System (datum is D_OSGB_1936) with units in metres.

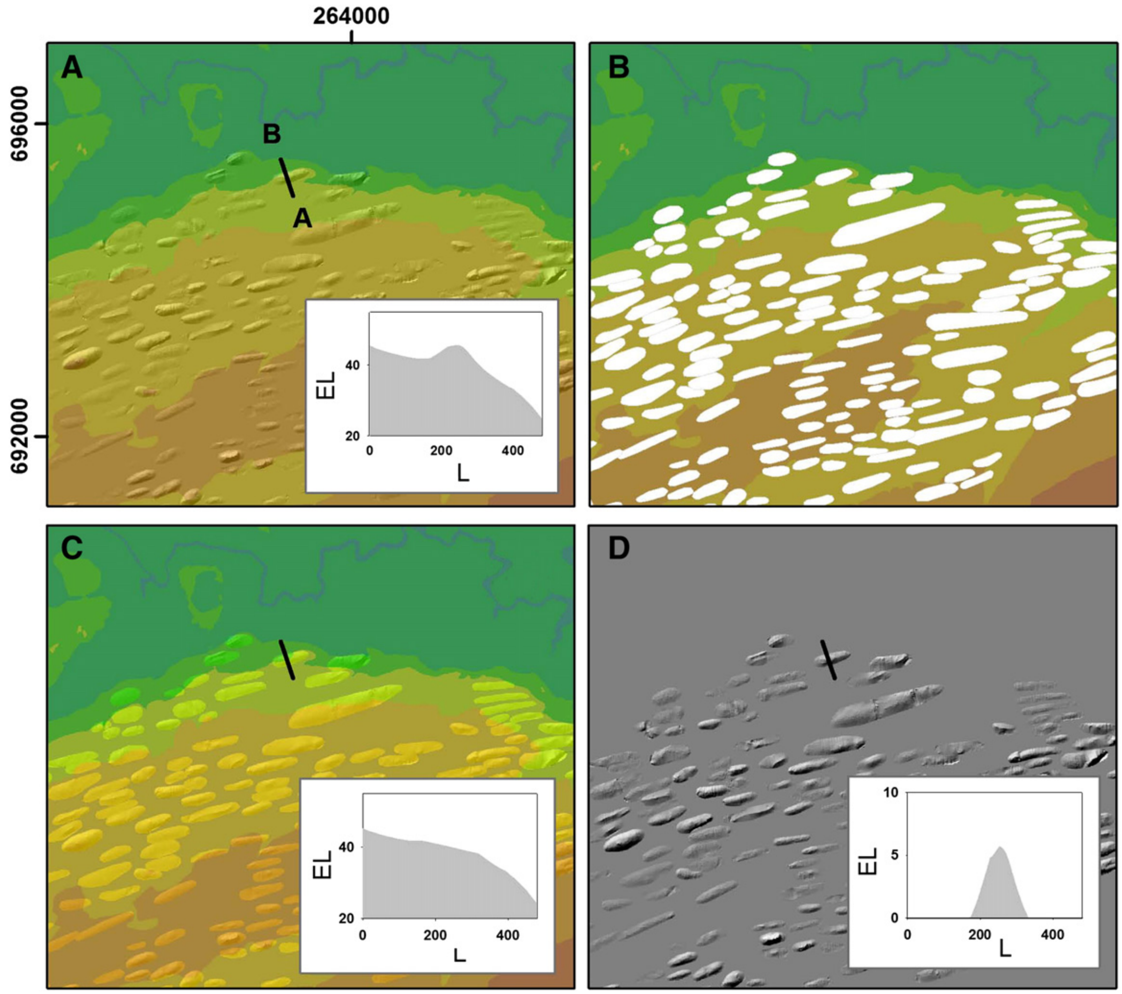

Fig. 4. An example (from the Glasgow area) of the GIS technique applied to evaluate drumlin relief. The simple DTM of the area (colours graded to elevation) with an inset showing a (transverse) topographic profile across one of the drumlins, from A to B (units are in metres) (A); the same DTM without the drumlins (B); the original DTM with a planar surface interpolated in the drumlin spaces: the 'drumlin-less DTM' now showing a planar surface that corresponds to the drumlin base (C); the subtraction between the original DTM and that of case (C) that yields a flat zero surface (green) above which stand all mapped drumlins with their estimated relief; the topographic profile across the same drumlin now shows zero values before and after the drumlin along the profile and the correct elevation of the drumlin, 6 m in this case (D). Coordinates refer to the British National Grid System (datum is D_OSGB_1936) with units in metres.

2009). This has the potential of improving the interpolation of the drumlin-less surface whenever irregular elements are present around the edge of the landform. A typical case is that of a field boundary wall that crosses a drumlin: this might result in a drumlin-less surface slightly higher than it should be. In this paper however, by deriving the TIN from a drumlin outline so closely sampled (every 5 m), this problem is essentially overcome: the presence of a wall or any other irregularities would only affect an extremely limited portion of the newly derived drumlin-less surface. Also, this is unlikely to happen exactly at the point where the maximum drumlin relief occurs and even when this happens, the slightly mistaken drumlin relief value would be averaged out by the large sample size statistics.

(ii) *Generate a DTM of just the drumlin upper surface, with no underlying landscape*. By subtracting the elevation values of the drumlin-less DTM from the original land surface DTM, a new DTM is obtained, called here "*Δ_DTM*" (Fig. 4D). All background (non-drumlin) pixel values should now be zero, while all drumlin pixels have a positive number (roughly representing drumlin relief, see next point). However, two small problems emerge. Firstly, some negative pixel values were encountered; this is owing to the fact that locally a drumlin could contain artificial or natural excavations from erosion: a road cut, a quarry, the cut of a river, a landslide, etc. If these 'cuts' are below the conceptual base of the drumlin, then negative pixels emerge. For such cases, this method of calculating drumlin relief might be inappropriate and, to be conservative, we decided to exclude from the analysis all those drumlins that contained >25% negative pixels. Secondly, in our mapping database of drumlins ($n = 36{,}032$) it was common to find either completely superimposed or slightly overlapping drumlins for which our technique might produce spurious results. To discount both these effects from the analysis, we filtered out the 'problematic' drumlins and this resized the database down to 25,848 drumlins.

(iii) *Identify the relief for each drumlin.* From the new *Δ_DTM* derived above, it is simple to automatically identify the pixel of highest value within each drumlin, which represents the point of the longest vertical distance between the drumlin surface and its base, i.e., (a) in Fig. 3. This vertical distance represents drumlin relief for those drumlins lying on a perfectly horizontal landscape. Otherwise, the exact drumlin relief (h in Fig. 3) could be easily derived from a by applying simple trigonometric laws: h is equal to a times the sine of the angle opposite to h. This angle is equivalent to the difference between 90° and the angle of the

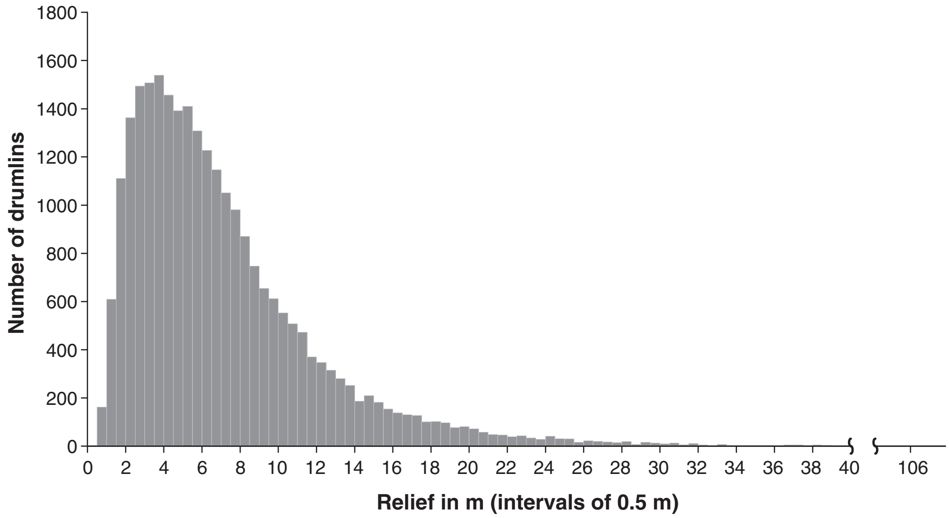

Fig. 5. British drumlin relief frequency with relief classes interval of 0.5 m. Some 41% of all drumlins are <5 m and 79% are <10 m in relief. The frequency rises quickly to the mode value and decreases gently towards the maximum values of drumlin relief.

drumlin base surface (θ in Fig. 3), which is a parameter that was easily measured from the original drumlin-less DTM.

This three-step procedure was applied to all drumlins of the filtered database, and the final results were then analysed statistically to derive the minimum and maximum values of drumlin relief as well as the mean value with its standard deviation and the mode. The correlation coefficient between drumlin relief and drumlin length, width, elongation (computed elsewhere for the same drumlins and described in Clark et al., 2009), and area was evaluated.

4. Results

4.1. Drumlin relief statistics

The quality controlled database of 25,848 landforms revealed a drumlin relief mean value of 7.1 m and a standard deviation of 5.3 m. Interestingly, and to confirm the hypothesis that sample size outweighs any precision issue, the same analysis was conducted over the unfiltered database of 36,032 drumlins where the mean drumlin relief is 6.7 m, just 4 dm lower than that of the filtered database. The frequency distribution is remarkably smooth, indicative of a sufficient sample size (Fig. 5), and shows an abrupt rise in the frequency of drumlins between 0 and 3 m height, followed by a fairly constant decrease from the modal class (3.5 to 4 m) towards the maximum values. The distribution has a high positive skewness of 2.6. Although the range of drumlin relief is relatively large, the most common height (mode) occurs at around 4 m. Some 41% of all drumlins are <5 m and 79% are <10 m in relief. While 0.5 m may be considered as the minimum drumlin relief value (with 163 drumlins with a relief between 0.5 and 1 m), defining an exact maximum value of drumlin relief is more difficult. Few drumlins are higher than 40 m and the decaying frequency of drumlin relief approaches zero at around 30–40 m (Fig. 5). In fact, we suspect that the few landforms mapped as drumlins that have an even larger relief (up to a maximum of ~100 m) are likely to represent bedrock hills smeared by drift, but only an ad hoc and time-consuming investigation on the internal structure of these hills would be able to confirm our suspicion. For these reasons, of the various statistics presented here, we regard the maximum drumlin relief as the least robust.

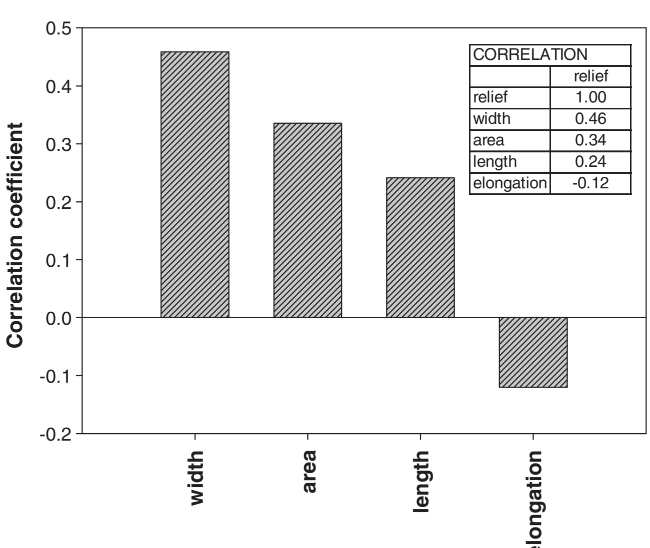

Fig. 6. Correlation coefficients between drumlin relief, width, area, length, and elongation. Bars are ordered from the highest to the lowest absolute correlation. All are statistically significant because of the large size of the database.

4.2. Comparison with drumlin altitudinal range

Mean altitudinal range, which is how drumlin relief was classically measured (i.e. Fig. 3), of the 25,848 British drumlins is 22 m (standard deviation of 14.7 m), three times as much as the mean value for drumlin relief measured with the new technique suggested in this paper. Values of altitudinal range vary between 1.3 and 229.5 m.

4.3. Relationship with other morphometric variables

The ratio between drumlin length and drumlin relief (l/h) is characterized by an average value of 123, i.e. drumlins are 123 times longer than in height, and 80% of the drumlins have l/h values between 36 and 253. The average ratio of drumlin width and relief (w/h) is 42, and 80% of the drumlins have w/h values between 16 and 81.

The linear correlation coefficients (Pearson r) between drumlin relief, length, width, elongation, and area are all relatively low (Fig. 6), although they are all statistically significant because of the large size of the database. The highest correlation is between drumlin relief and width (0.46), followed by the area (0.34). The lowest linear correlation is with the elongation (-0.12). Other types of correlations (i.e. nonlinear) were also tested, but none showed significantly higher coefficients.

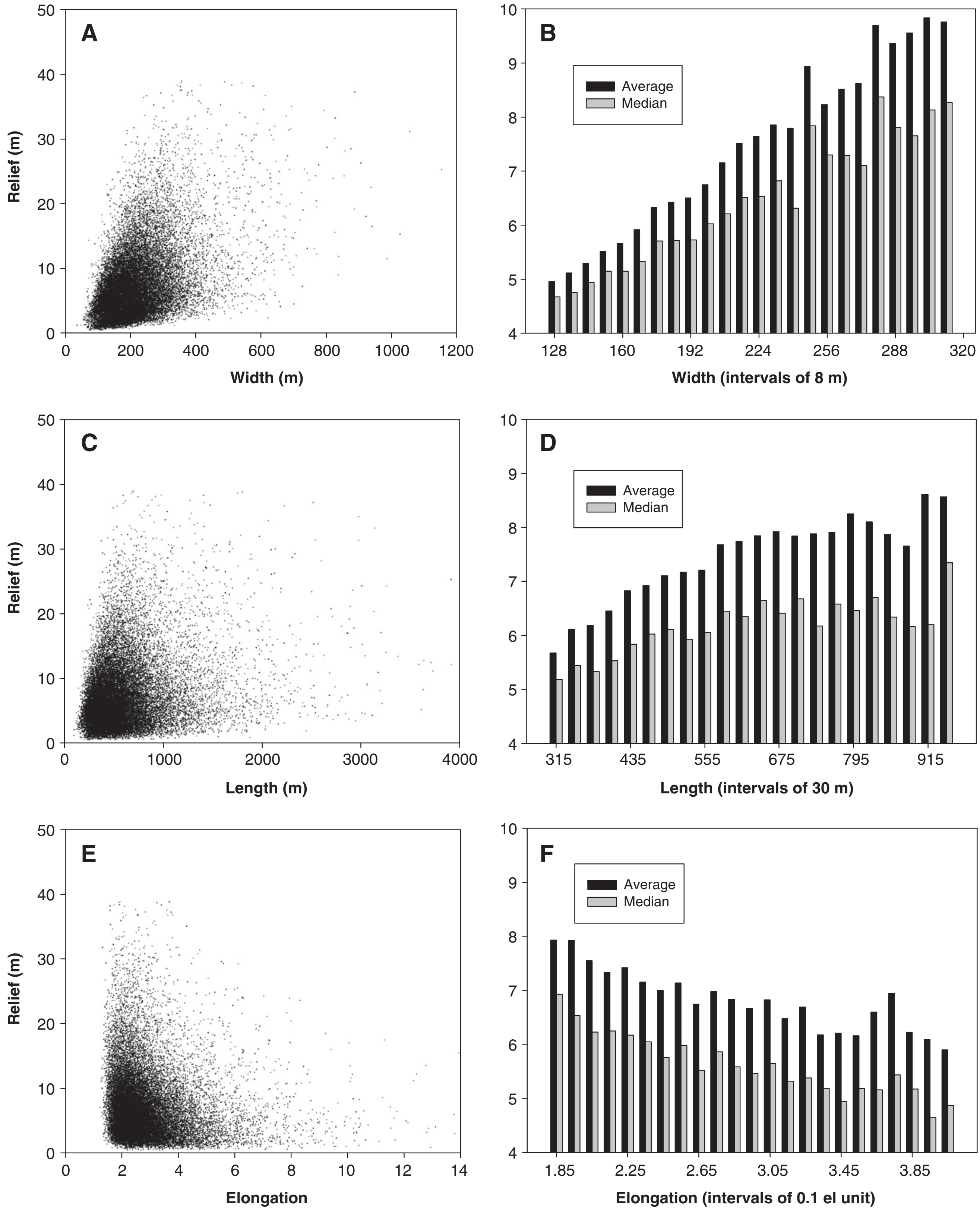

Fig. 7. Drumlin relief plotted against width (A), length (C) and elongation (E): each point represents one drumlin, $n = 25{,}848$. Note in the relief vs. width plot (A) the wedge-shaped cloud of data points has relatively clear lower and upper boundaries. Wider drumlins can be shallow in relief (e.g. 600 m wide and only 5 m in relief), and they also show a much larger range of relief values. In plot (C), the relief range increases with the length for lengths lower than 1000 m. In plot (E) drumlin relief range diminishes for increasing values of elongation. Plots (B), (D), and (F) show the average and median values of relief per given class of width, length, and elongation, respectively. In order to make the graphs easier to read, values of width, length, or elongation below the 10 percentile and above the 90 percentile have not been plotted. Note how both the average and median values of drumlin relief tend to (i) increase with the width (B), (ii) increase but at a slower pace with the length (D) and (iii) slowly decrease with the elongation (F).

When plotted together, drumlin width and relief draw a wedge-like cloud of points delimited by relatively sharp boundaries (Fig. 7A). In other words, for growing values of width, drumlin relief minimum and maximum spread increases. In particular, while the minimum boundary grows at a slow pace, being always extremely close to the minimum possible drumlin relief, the maximum boundary grows much faster. As a result, wider drumlins exhibit a much wider range of relief than do narrow drumlins; and in general, relief does not exceed 7% of the width. Overall, when the analysis focuses on the 80% of the data (10 and 90 width percentile interval) and the average and median values of relief are calculated per class of width (Fig. 7B), we found a strong tendency for the relief to grow with the width. Although characterized by a lower linear correlation, drumlin relief and length (when plotted together) also show some interesting general trends (Fig. 7C). In particular, the maximum possible drumlin relief for a given length grows proportionally with the length. In general, for $l<1000$ m, drumlin relief does not exceed 4% of drumlin length. Also, the average and median values of relief tend to increase as the length increases (Fig. 7D), although not as much as they did with the width. Finally, the plot of drumlin relief and elongation (Fig. 7E) shows that the maximum drumlin relief for a given elongation diminishes with the elongation so that more elongated drumlins exhibit a much narrower range of relief values than do less elongated drumlins. When the average and median values of relief are plotted against the elongation (Fig. 7F), the relief tends to diminish with the elongation.

5. Discussion

5.1. Possible caveats

The data represents the first widespread analysis of drumlin relief over a large sample size of tens of thousands of landforms, and the smooth frequency distribution of Fig. 5 (despite a very narrow bin interval of just 0.5 m) suggests that this size is adequate. However, it is important to remember the caveats earlier mentioned (e.g., Fig. 1) regarding what these relief measurements actually represent: essentially the distance between summit and base, this latter defined by the bounding break-of-slope. If the most appropriate model of three-dimensional drumlin geometry is that of a continuous surface of waveforms and our mapped break-of-slopes are merely a consequence of inter-drumlin sediment infills, then our relief measurements would be some fraction of the true relief (or amplitude). Knowledge of sediment thicknesses across the areas of investigation is insufficient to be able to directly address this issue. However, it is possible to look at it in a more general way by asking what the typical sediment thickness in drumlin fields is and how this compares with the drumlin height metrics. The British Geological Survey (BGS) has compiled a UK Superficial Deposits Thickness Model that defines maximum thickness of Quaternary sediment or 'drift' (i.e. till and any post-glacial sedimentation, in our study area) and is available at a resolution of 50 m. The database is a blunt instrument for assessing the thickness of inter-drumlin areas in the present form because (i) it is built from a series of non-uniformly distributed point sources (boreholes) across the region, (ii) the minimum value is always set at 1 m (for all areas covered by Quaternary deposits) thus discarding anything lower, (iii) it contains a minor percentage of very high values (up to 300 m) that largely influence any average analysis and (iv) drumlins (and inter-drumlins) are not discernible as they have effectively being averaged as a consequence of the sparse boreholes. However, it represents the only available option and it is a useful guide for estimating the typical sediment thicknesses across the glaciated region of the UK. From this database, the drumlinised terrain (which includes the mapped drumlins and a 1 km buffer around them) is characterized by an average maximum sediment thickness of 6.9 m, but excluding the seemingly anomalous high values this reduces to 4 m. However, looking at the frequency distribution, it appears that most (52%) of the drumlinised terrain is characterized by a maximum sediment thickness of less than 2 m. As all these estimates are a combination of the drumlins themselves (whose average maximum thickness is 6.7 m) and inter-drumlin areas, it is unlikely that the thickness of sediment in the inter-drumlin areas is sufficiently thick to be a significant problem in the estimates of drumlin relief. As highlighted through the analysis of other metrics (e.g. Clark et al., 2009; Spagnolo et al., 2010), we presume that British drumlins are not special, such that this sample is representative of the much wider global population. However, this is just an assumption as the analysis of drumlin relief at this level of detail requires a high resolution DTM, which is not yet available or easily accessible for other regions. In conclusion, the drumlin relief metrics presented in this paper appear to be a reasonable proxy for the relief of the drumlins generally.

5.2. Drumlin relief statistics

The first and most striking result of our analysis is that drumlins are very shallow landforms indeed. To our great surprise we found that the most commonly occurring drumlin relief is only 3.5–4 m, and almost 2000 mapped drumlins (7% of the total) have a relief of <2 m. For features hundreds of metres in length and width, this is very shallow. Such was our astonishment at this result that we questioned the methodology and checked the analysis again and then undertook field visits on some low relief individuals. As an example, a drumlin mapped and identified by our method as being 2.1 m in relief (and 430 m long and 150 m wide) was visited (Fig. 8). Such is its subtlety and shallow slopes it is unlikely that this example would have been noticed and mapped as a drumlin by field investigation or a coarser DTM; but the high resolution DTM used here (Fig. 9A) revealed it to be a drumlin, and the slopes and shape in the field confirm this. In order to double-check we also acquired an even higher resolution DTM (2-m horizontal resolution and centimetric vertical resolution) for the specific area, collected with LIDAR techniques (Fig. 9B), and this also confirmed the presence of a drumlin in this position. We conclude that the DTM used here, when processed with a vertical exaggeration (e.g. ×4), is indeed capable of being used to identify and map drumlins of as little as 1 or 2 m and that our mapping and relief estimates are acceptable.

Along with the low mode, the mean relief is also surprisingly low, only 7.1 m. This is certainly much lower than the figures of some tens of metres normally reported in textbooks and almost half as much as the weighted mean drumlin relief (13 m) compiled from the literature ($n=13{,}427$) (Table 1). Similar low relief values were found by Jauhiainen (1975) as well as Wysota (1994) in various Polish and German sites. However, if we compare our result with those of the three largest databases ever analysed before, all relief mean values are between two and three times higher than those found in Britain (Fig. 10). A possible reason for these discrepancies is that drumlins in different regions are actually different; but given the wide area of study and range of ice sheet contexts, topographic settings, and rock lithologies covered by the British database (Fig. 1), we find this unlikely. The more probable explanation is with regard to the data sources used for mapping. For instance, Zakrzewska Borowiecka and Erickson (1985) analysed drumlin relief on 1:24,000 maps with a contour interval of 12 m; Francek (1991) used 1:24,000 maps with contours every 3 m; Wysota (1994) worked on 1:10,000 maps; Hättestrand et al. (2004) mapped from aerial photos with 4.8-m horizontal resolution, but they could only map drumlins higher than "*a few metres*" and they measured drumlin relief on 10-m contour maps. In summary, none of these studies had the chance of measuring drumlin relief with a vertical accuracy as good as the 0.7–1 m of the DTM used here. More importantly, in all mentioned studies drumlin relief was overestimated because the altitudinal range of the landform was measured instead of the relief as defined in this paper (Fig. 3). As a comparison, the altitudinal range of British drumlins produces a mean value of 22 m, three times larger than the measured British mean drumlin relief.

Fig. 8. Drumlin with a relief of 2.1 m near Kaber, Eden Valley, Cumbria (A) and the same with a dotted line indicating its cross-profile positioned transverse to ice flow direction (B). The picture is taken from the top of the drumlin looking downstream along its crest. Notice the sheep flock in the middle as scale reference; they also give a sense of how shallow this barely visible drumlin is (UK OS grid reference NY804117).

This relatively large difference between the mean relief and the mean altitudinal range in Britain deserves further examination. One might be tempted to conclude that this is owing to a high percentage of drumlins lying on steep hillslopes. However, the mean angle of the 36,032 drumlin base surfaces, as measured as part of our technique, is only 3.2° — and only about 20% of the drumlins lie on a surface steeper than 5°. The reason why the mean altitudinal range is much higher than the mean relief is that many of the mapped drumlins, although lying on a gentle hillslope, are relatively long, such that even on gentle slopes the errors rapidly accrue. For example, a 1-km-long and 10-m-high drumlin lying on a 5° hillslope would yield an altitudinal range of up to 80 m.

5.3. Implications for drumlin evolution

Regardless of specific formation theories, there are at least five different ways drumlins can reach their present-day relief:

1. All drumlins within a field were initially small and grew to their present size.
2. All drumlins within a field were large but shrank (were eroded) to their present size.
3. A combination of the above with drumlins fluctuating in size at various growth and shrinkage phases during an ice flow event or even being modified over multiple glaciations
4. Individual drumlins within a field evolved independently to their present size by progressive growing (and/or shrinking) so that the observed drumlins represent a mixed population of individuals, some that are preserved shortly after they had formed and others that are highly evolved (i.e. the full range from babies to old people rather than just the average).
5. Drumlins did not evolve but were "printed" on the landscape, by some form of template.

The linear correlation between relief and all other parameters (width, length, elongation, and area) is generally weak (<0.5). If drumlins are printed landforms, this means that they were not formed as 'plaster casts' by a single process, with their size simply depending on the intensity of the process. In other words, there is no standard drumlin size or aspect ratio, and drumlins could be 'printed' in a wide range of sizes. If, however, drumlins are growing phenomena, then the low correlations between relief and the other parameters indicate that drumlins do not grow isometrically but much more in length and width than in relief. On the contrary, if drumlins shrank to their present size, then the erosion worked faster on the top of the drumlin than at its flanks.

Of all correlations, that between relief and width is the strongest (0.46) (Fig. 6). This largely confirms the findings of the earlier investigations (e.g. Mills, 1987) and indicates that we should seek a process-based explanation for why width and relief scale together much better than does relief with length or elongation. We also noticed that drumlin

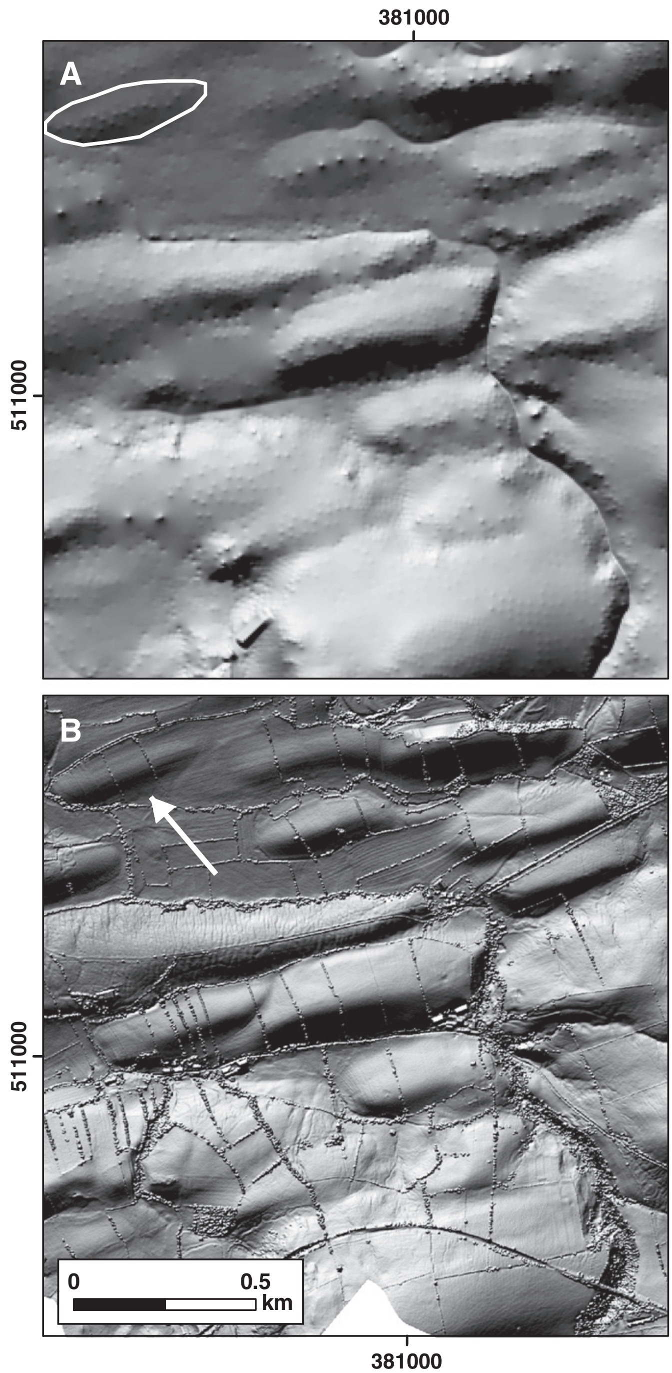

Fig. 9. Appearance of the 2.1-m-high drumlin near Kaber, Eden Valley (Cumbria), shown in Fig. 8 on different resolution DTMs: from the 5-m resolution NextMap Britain DTM (InterMap Technologies © BGS(NERC)) (A) and from the 2-m LIDAR (LIDAR data Copyright Geomatics Group 2008) DTM for the same area (B). Drumlin is identified by a white outline in (A) and an arrow in (B). Note how easily this drumlin appears, especially on the LIDAR image (B), although it is only a few metres in relief. Coordinates refer to the British National Grid System (GCS_OSGB_1936) with units in metres.

maximum, average, and median relief tends to increase as the width gets larger (Fig. 7A and B). This indicates that when drumlins are wide enough, their relief could become large. However, when their width is small, the relief is now forced to be extremely small. If drumlins are growing/shrinking phenomena, this relationship between relief and width could be interpreted in 'aerodynamic' terms. In particular, if drumlins do not have a sufficiently wide base, an excessive relief would result in a too large drag against the flow of the ice without a proper supportive base. Drumlins would not be able to stand against the ice flow and pressure, and they would be torn off or lowered (eroded on the top) to a more acceptable relief or entirely removed. As a tree, for example, needs large roots to develop a large crown able to resist the disruptive action of the wind, or electricity pylons require broadly enough spaced cement bases to sustain their height, drumlins need a large base to grow in relief against the flow of ice.

Although less pronounced, drumlin relief tends to increase as the length gets larger (Fig. 7C and D). This is in contrast with the hypothesis that drumlins represent attenuated landforms that become progressively longer at the expense of their relief (e.g., Piotrowski, 1989), assuming drumlins are growing phenomena. If sediment is removed from the drumlin summit to be redeposited at its far (lee) end, a negative correlation between relief and length should be expected. Our results indicate the opposite and suggest that if drumlins are a growing phenomenon they do not need to use the sediment from their own summits in order to increase their length. Finally, an apparent contradictory result is that drumlin relief appears to diminish as the elongation increases (Fig. 7E and F). Elongation is the ratio between length and width, so that higher values of this parameter mean longer and narrower drumlins. From Fig. 7 we know that longer drumlins tend to be slightly taller, while narrower drumlins tend to be significantly shallower. Clearly, of the two parameters, it is the effect of the width, with its stronger correlation with the relief, that can be most seen in the relief response to increasing values of elongation.

6. Conclusions

This paper reports a systematic survey that applies a newly developed GIS technique to extract relief data from 25,848 drumlins mapped on a high resolution DTM in Britain. Mean drumlin relief (7.1 m) is found to be around half of that reported in the literature, and to our great surprise we found that the most commonly occurring relief is only 3.5 to 4 m and with some 7% having a relief <2 m. For features hundreds of metres in length and width, this is very shallow indeed (average length/relief aspect ratio of 123) and we thus now know that most map and field-based investigations were unable to detect the (dominant) small drumlins because of scale limitations. For theories wishing to explain drumlin formation, the target relief is now much lower than usually reported; and the relief statistics contained herein should provide a useful test of model predictions. Could theory explain why most drumlins only reach 3.5–4 m in relief?

Correlations of relief against length, width, and elongation were all found to be low (<0.5) and with relief-width as the strongest (0.46). This demonstrates that drumlins are not 'printed' on the landscape with a fixed proportion of the parameters and suggests a complex formation and evolution of these landforms. They do not evolve (by growing and/or shrinking) into their present-day size in an isometric way: they either grow much faster in length and width than in relief or they are much more eroded on the top rather than from the sides. If drumlins are a growth phenomenon, then these data imply that length and elongation are merely a consequence of the history of development (and perhaps ice velocity), but relief and width are the fundamental elements of the drumlin-process. Importantly, for model predictions, the scatter of data points in a relief against width graph displays a crisp upper boundary indicating strong natural limits for the growth in relief of a drumlin related to the width and which require explanation or simulation. Wide as well as narrow drumlins could be shallow in relief, but most drumlin relief does not exceed 7% of their width, and as drumlin width increases so does the possible maximum relief. Another important trend that may be used to test models is that drumlin relief tends to increase as the length increases. This is also relevant because it indicates that drumlins do not become progressively more elongated by subtracting sediment from their summit.

Finally we note that the common description of drumlins as resembling half-eggs is somewhat misleading, especially with regard to their relative relief. For the mean length drumlin of 629 m (from Clark et al.,

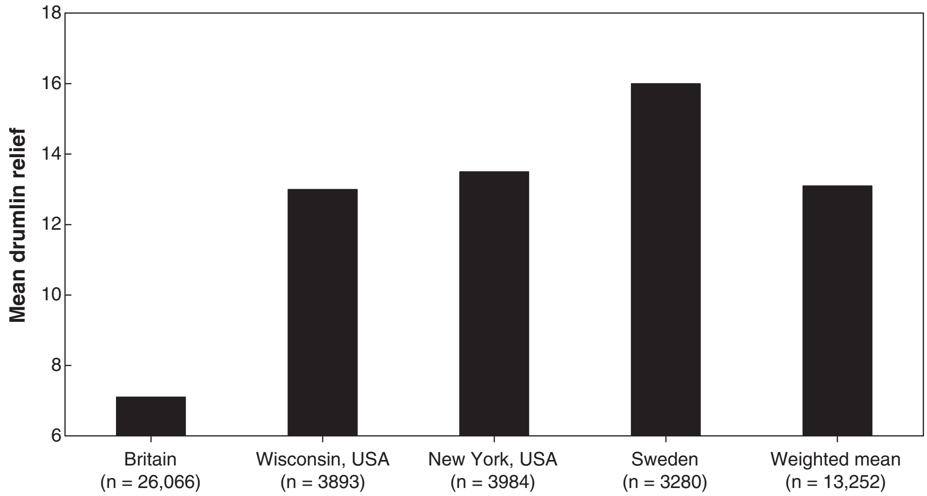

Fig. 10. Comparison between mean drumlin relief from the British database (present work) and the same parameter analysed in Wisconsin (Zakrzewska Borowiecka and Erickson, 1985), New York State (Francek, 1991) and Sweden (Hättestrand et al., 2004). The last column refers to the weighted mean drumlin relief from all papers analysed in the literature review (see Table 1 for exact numbers). The difference between the British versus other areas arises not from real differences in the drumlin relief but from the measurement technique and the data sources (see text).

2009), if it was actually half-egg shaped, it would have a relief of a staggering 239 m, which is higher than any drumlin ever recorded and much higher than the usual 3 to 25 m range in relief that we found to typically correspond to this length.

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